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Unhealthy Love Dynamics Stimulates Dishonest Interactions Playing A Leading Role in Divorce: A Model Based Mathematical Approach

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Abstract

Love dynamics indicates to the designs of interaction, behaviours and emotional connections within a relationship. These dynamics can be healthy or unhealthy. Unhealthy love dynamics comprises power imbalances, lack of communication, sensitive manipulation or a lack of respect. Any kind of unhealthy love dynamics always favours dishonest communication, which promotes marital unrest and ultimately results in divorce. In our research paper, we have discussed the impact of dishonest interaction between a Single person and a Married person, which plays a foremost role in divorce. We have tried to establish that the influence of Single people's dishonest connections on the lives of Married people will decrease as the time goes on.

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Keywords: Single, Married, Unmarried, Divorced, Love Dynamics, Dishonest Interaction.

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Introduction

The importance of love dynamics in determining the depth of a relationship is immense. There are several components of love dynamics by which we can analyse a relationship. Components are communication, trust, respect, emotional support, conflict resolution, passion, intimacy, commitment etc. Love dynamics are not static at all. It can always be changed and developed as relationships progress through different stages and challenges. Healthy love dynamics involves open communication, mutual respect, trust and support for each other, whereas unhealthy love dynamics includes imbalances, lack of communication and emotional manipulation, which gradually breaks the bonds of a healthy marital relationship and leads to divorce.

Marriage is the gateway to family life in any community, regardless of race, religion or caste. Wedding is a socially recognised association, in which couples dedicate themselves to each other with the desire for an everlasting intimate relationship. Marriage is a legally accepted bond and divorce is the legal process of severing that bond. Divorce is an authorised procedure between Married couples by which their marital relationship comes to an end. On the other hand, divorce, which is a complex procedure that can be done by mutual consent or through the courts, terminates the legal rights and responsibilities of a Married couple. This includes property division, child custody, alimony (financial support that a person is ordered by court to give to spouse during separation) and other related issues. There are many reasons for divorce that don't always come to the fore. Yet, some of the common issues are lack of mutual understanding, extramarital affairs, domestic violence, drug addiction, financial problems etc. Divorce rules vary from country to country, but the divorce process is usually completed in court through a legal process. However, regardless of the country, conflict and separation devastate people socially, economically and above all psychologically. In many cases, people lose their social balance and become mentally as well as economically unbalanced. In one word, just as marriage tries to lead a person towards a stable life, divorce pushes a person towards an unstable life.

Divorces have become a common fragment of American life, affecting offspring of all traditional backgrounds, religions and social position. Divorce rates have risen day by day in England and Wales. A discrepancy in marital satisfaction as well as the economy has a noteworthy influence on divorce in Spain. Divorce is a very common phenomenon in Africa with instant and long-term significance. The rate of Divorce among young people have been increased in South Africa, which has one of the highest rates in the world. Divorce has broader societal, cultural, economic, psychological and political penalties on stable families to destabilise. In some countries, Divorce is relatively easy and common, while in others it can be difficult for religious or traditional reasons. Where marriage is considered a sacred bond, divorce is discouraged. However, in any case, if a couple decides to divorce, they should seek legal advice (Mekonnen et al., 2019; Mikolai & Kulu, 2018; Kleinsorge & Covitz, 2012; Tesseman et al., 2022).

Alfred Hugo and Eva M Lusekelo (2021) engrossed on the dynamics of Marriage and Divorce scenarios in their research article. They applied counselling as the probable regulator mechanism to preserve armistice and synchronisation among Marriages. They established that increasing psychoanalysis effort diminishes the adversity lifestyle and intricacy within Married and also Divorced individuals. Iortyer Gweryina formulated an epidemiological model of Divorce by means of normal occurrence function as force of marital divergence. They proved Anti-Divorce Protocols and resolution can cooperatively stabilise Marriages. They also established Married cases that endure from Divorce epidemic in 30 years span of Marriage couldn’t interrupt (Gweryina et al., 2021). Michelle Moon (2011) examined the hypotheses concerning the influence of marital standpoint on parents’ attitudes in the direction of the effect of divorce on their children. Tessema et al. (2022) expressed a Mathematical Model for Divorce in Marriage and prolonged into an Optimal Control Model. They illustrated the sensitivity analysis of the model. An Optimal Control Model was established by integrating three–time dependent Control strategies (Couple Relationship Education, Reducing getting Married at too young age and Consulting Separators to recommence their Marriage) on the deterministic model (Tessema et al., 2022). Berat Karaagac and Kolade M. Owolabi formulated the Mathematical Model of Divorce in the light of the Fractional Caputo–Fabrizio derivative. Fractal analysis of Divorce, which intensely wounds the society, brightens the impetus for their research paper (Karaagac & Owolabi, 2023). Padder et al. (2025) introduced a model based on Ordinary Differential Equations (ODEs), with statistical hypothesis testing to inspect Divorce inclinations over two decades

with the help of longitudinal and actual facts and figures. They confirmed Robustness through Residual Analysis, Durbin - Watson, Jarque - Bera Statistics and Normality Testing. To keep in mind the limitations of the deterministic method owing to the haphazardness of system behaviour, a more accurate stochastic epidemic model of Divorce dynamics was considered by Shah et al. (2025). They recognised that the Broken individuals improved from their struggle and re-joined to its Married regime.

In our research article, we have focused on the connection or interaction between Single and Married people among the many causes of divorce. It is undeniable that this connection is never made in an honest way. It is not always visible or understandable from the outside. That is why this dishonest connection sneaks into people's daily life. It gradually has a serious impact on couple's livelihoods. Although it is also true that dishonest communication between a Married person and another Married person can be a cause for divorce. A Married person can also create problems in the life of another Married person. It may cause breakdown in their conjugal life. But we do not bring that issue into the scope of our research paper, as the inclusion of a Single person in the life of a Married person can have a greater impact than that. Third-party intrusion does not always have to be in the outside world, but can sometimes occur inside the home as well. Although this is true, this phenomenon is very common in the outside world, so it is not as noticeable on Social Media, Television or Newspaper. But this type of incident that happens inside the house can easily spread around. The main reason for this is that the rate of such incidents occurring within the home is relatively low. The focal theme of our article is to reduce any form of improper third-party communication. After getting Divorce, some portion of Divorced population become mentally imbalanced. Later, if the problems become more severe, they are taken to a mental health center or mental hospital for proper treatment. Since they are unable to return to the Single population after Divorce, that population is being disconnected from our system. In our research paper, we have considered three types of mortality rates for Single, Married and Divorced people. The mortality rate of Single people is higher than that of Married people. Because after a certain age, the lives of Single people become unstable. They are more likely to get sick or die because they don't have anyone to look after them or take care of them. A frustration among Single people is mainly due to not being able to establish love or one could say, not being able to find a suitable life partner. This incident increases the tendency of suicide among them. But generally Married people live a well-disciplined life. Their discipline is the key to their longevity. Although many people choose to commit suicide due

to marital discord or depression due to marital problems. Everything has exceptions, we are not capturing those exceptions in our research. Next, let's talk about Divorced people. We have already discussed that a portion of Divorced people become depressed. Another class of them becomes accustomed to living a chaotic life due to separation. This life practice is quickly leading him towards death. The remainder reverts back to a Single population. We have accepted the marriage cycle in our research paper to be a maximum of two times. A portion of those people prepare themselves for marriage again for second time. In a second marriage, a Divorced person can marry a Divorced person, also Divorced people can marry Single people too. We do not distinguish between their tendencies.

Here we assume the Unmarried and Single individual in one category, though they belong to two different groups. The presence of a Single individual can create problems for a Married couple in many cases, but it depends entirely on the situation. If the Single person creates a lack of intimacy or trust between the couple or creates suspicion and distrust in their relationship, then this can definitely be a problem. However, if they have a healthy and respectful relationship, the presence of a Single person may not be a problem. But in many cases, even if it is not a problem at first, it is normal for problems to arise later for the Married life of a couple. For the case of trust and privacy, if a couple is not faithful to each other or has trouble maintaining privacy in their personal lives, the presence of a third person can create more distance between them. If an Unmarried or Single person develops a close friendship with someone in the couple, it can cause problems in their Married life. In some cases, the presence of an Unmarried person can also create problems due to societal pressure or traditional beliefs. In most cases, their relationship is not considered normal in the eyes of society.

The Basic Assumptions and the Formulation of Mathematical Model

The entire population is divided into three subpopulation, those who reach at the age of getting Married are Single individuals, denoted by S(t), Married individuals are denoted by M(t) and those who are Divorced, are denoted by D(t) We do not divide any of three subpopulation into two categories: Male and Female. We do not include any separate category of widows or widowers in our study. We have assumed that all of them would belong into the Married group. Here, α is the recruitment rate of individuals being Single in a particular geographical region, when he or she reaches at the age of getting Married.

Individuals are married at rate of γ and we assume $\in$ as the rate of Divorce. In our research article, we consider the most crucial parameter ξ as the dishonest interaction rate between a Single and a Married individual. The Divorced people join to the Single subpopulation at a rate β. Some portion of them again prepare themselves for second marriage at a rate β_1. Due to Divorce, some section of Divorced individuals becomes mentally imbalanced at a rate ϕ and they need to be taken care at either home or mental hospital. The mortality rates of Single, Married and Divorced population are denoted by μ_1, μ_2 and μ_3 respectively. Here, we assume that sex, race and social status do not affect the probability of getting Divorced. Keeping in mind the above assumptions, the mathematical model is formulated by the following system of Differential Equations (Murray, 2007):

$$\begin{aligned}
\frac{dS}{dt} &= \alpha - \gamma S + \beta D - \beta_1 DS - \mu_1 S \\
\frac{dM}{dt} &= \gamma S - \in M - \xi SM + \beta_1 DS - \mu_2 M \qquad (1) \\
\frac{dD}{dt} &= \in M - \beta D + \xi SM - \phi D - \mu_3 D.
\end{aligned}$$

where S(0) > 0, M(0) > 0 and D(0) > 0.

Theoretical Study of the System

First of all, we consider the system of equations (1) for positivity. Here, $\alpha - \gamma S + \beta D - \beta_1 DS - \mu_1 S = 0$ implies $S = \frac{\alpha + \beta D}{\gamma + \mu_1 + \beta_1 D} > 0$, $\gamma S - \in M - \xi SM + \beta_1 DS - \mu_2 M = 0$ implies $M = \frac{S(\gamma + \beta_1 D)}{\in + \xi S + \mu_2} > 0$ and $\in M - \beta D + \xi SM + \phi D - \mu_3 D = 0$ implies $D = \frac{M(\in + \xi S)}{\beta + \phi + \mu_3} > 0.$

Therefore three sub populations (Single, Married and Divorced) are always any imposed condition.

Here we assume that the mortality rates of Single, Married and Divorced population, denoted by μ_1, μ_2 and μ_3 respectively is in the relation $\mu_2 < \mu_3 < \mu_1$. Also, from our analysis, we can say $\mu_2 < \mu_1 < \frac{\alpha - \gamma S + \beta D - \beta_1 DS}{S}$. Moreover, it is stated that $\mu_2 < \frac{\gamma S - \in M - \xi SM - \beta_1 DS}{M}$ and $\mu_3 < \frac{\in M - \beta D - \xi SM - \phi D}{D}$.

From the system of equations (1), we can conclude that $\alpha > \mu_1 S + \mu_1 M + (\phi + \mu_3)D$ for the existence of the system. We can derive two equilibrium system of equations. One is Divorce-free Equilibrium Point (S. M. 0) and other is Interior Equilibrium Point (S*. M*. 0*) where $S^* = \frac{\alpha + \beta D^*}{\gamma + \mu_1 + \beta_1 D^*} > 0, M^* = \frac{S^*(\gamma + \beta_1 D^*)}{\in + \xi S^* + \mu_2}$ and $D^* = \frac{M^*(\in + \xi S^*)}{\beta + \phi + \mu_3} > 0$. In our research article. We have discussed with Divorce-free system as we are trying to eradicate or atleast minimise the Divorce population. The Jacobian matrix for the system of equations (1) is given by.

$$J(S.M.D) = \begin{pmatrix} -\gamma - \mu_1 - \beta_1 D & 0 & \beta - \beta_1 S \\ \gamma - \xi M - \beta_1 D & -\xi S - \in -\mu_2 & \beta_1 S \\ \xi M & \in + \xi S & -\beta - \phi - \mu_3 \end{pmatrix}.$$

The Characteristic Equation

The Jacobian matrix for the Divorce-free equilibrium point is

$$J(S.M.0) = \begin{pmatrix} -\gamma - \mu_1 & 0 & \beta - \beta_1 S \\ \gamma - \xi M & -\xi S - \in -\mu_2 & \beta_1 S \\ \xi M & \in + \xi S & -\beta - \phi - \mu_3 \end{pmatrix}.$$

The characteristic equation for the Jacobian matrix J(S. M. 0) is given by $|J(S.M.0) - \lambda I| = 0$, which is a cubic equation in A of the form $\lambda^3 + A_1\lambda^2 + A_2\lambda + A_3 = 0$. The coefficients A_1, A_2 and A_3 are found as follows:

- $A_1 = -\text{tr}(J)$.
- A_2 is the sum of the principal min ors of J.
- $A_3 = -\det(J)$.

Coefficient A_1

The trace of the Jacobian matrix J is the sum of its diagonal elements:

$$\text{tr}(J) = (-\gamma - \mu_1) + (-\xi S - \in -\mu_2) + (-\beta - \phi - \mu_3).$$

Therefore, A_1 is given by

$$A_1 = \gamma + \mu_1 + \xi S + \epsilon + \mu_2 + \beta + \phi + \mu_3.$$

Coefficient A_2

The coefficient A_2 is the sum of the principal minors of the Jacobian matrix:

$$\begin{aligned} A_2 &= \begin{vmatrix} -\xi S - \epsilon - \mu_2 & \beta_1 S \\ \epsilon + \xi S & -\beta - \phi - \mu_3 \end{vmatrix} + \begin{vmatrix} -\gamma - \mu_1 & \beta - \beta_1 S \\ \xi M & -\beta - \phi - \mu_3 \end{vmatrix} + \begin{vmatrix} -\gamma - \mu_1 & 0 \\ \gamma - \xi M & -\xi S - \epsilon - \mu_2 \end{vmatrix} \\ &= [(\xi S + \epsilon + \mu_2)(\beta + \phi + \mu_3) - \beta_1 S(\epsilon + \xi S)] \\ &\quad + [(\gamma + \mu_1)(\beta + \phi + \mu_3) - \xi M(\beta - \beta_1 S)] \\ &\quad + [(\gamma + \mu_1)(\xi S + \epsilon + \mu_2)]. \end{aligned}$$

Coefficient A_3

The coefficient A_3 is the negative of the determinant of the Jacobian matrix:

$$\begin{aligned} \det(J) &= (-\gamma - \mu_1) \begin{vmatrix} -\xi S - \epsilon - \mu_2 & \beta_1 S \\ \epsilon + \xi S & -\beta - \phi - \mu_3 \end{vmatrix} + (\beta - \beta_1 S) \begin{vmatrix} \gamma - \xi M & -\xi S - \epsilon - \mu_2 \\ \xi M & \epsilon + \xi S \end{vmatrix} \\ &= -(\gamma + \mu_1)[(\xi S + \epsilon + \mu_2)(\beta + \phi + \mu_3) - \beta_1 S(\epsilon + \xi S)] \\ &\quad + (\beta - \beta_1 S)[(\gamma - \xi M)(\epsilon + \xi S) + \xi M(\xi S + \epsilon + \mu_2)]. \end{aligned}$$

After simplification, we get:

$$A_3 = (\gamma + \mu_1)(\xi S + \epsilon + \mu_2)(\beta + \phi + \mu_3) - (\gamma + \mu_1)\beta_1 S(\epsilon + \xi S) - (\beta - \beta_1 S)[\gamma(\epsilon + \xi S) + \xi M \mu_2].$$

From R-II Criterion, we can conclude that the Divorce-free Equilibrium Point is stable only if $A_1 > 0, A_3 > 0$ and $A_1.A_2 > A_3$.

Expression for R_0

The Basic Reproduction Number R_0 is a threshold parameter that determines the stability of the Divorce-free Equilibrium. It is derived from the condition $A_3 > 0$. The expression for R_0 is found by setting $A_3 = 0$ which gives R_0 as follows:

$$R_0 = \frac{(\mu_1 \beta_1 S + \gamma\beta)(\epsilon + \xi S) + \xi \mu_2 M(\beta - \beta_1 S)}{(\gamma + \mu_1)(\xi S + \epsilon + \mu_2)(\beta + \phi + \mu_3)}.$$

The Divorce-free Equilibrium is locally stable if $R_0 < 1$ and becomes unstable when $R_0 > 1$.

The final expression for R_0 is presented in terms of the equilibrium values S and M for the sake of clarity and compactness. Substituting the full expressions for S and M, where $S = \frac{\alpha}{\gamma+\mu_1}$ and $M = \frac{S\gamma}{\epsilon+\xi S+\mu_2} = \frac{\alpha\gamma}{(\gamma+\mu_1)(\epsilon+\xi S+\mu_2)}$ given the equilibrium points:

$$S = \frac{\alpha}{\gamma+\mu_1} \quad \text{and} \quad M = \frac{\alpha\gamma}{\alpha\xi+(\epsilon+\mu_2)(\gamma+\mu_1)}.$$

The expression for R_0 becomes,

$$R_0 = \frac{(\alpha\mu_1\beta_1+\gamma\beta(\gamma+\mu_1))(\epsilon(\gamma+\mu_1)+\alpha\xi)}{(\gamma+\mu_1)^2(\alpha\xi+(\epsilon+\mu_2)(\gamma+\mu_1))(\beta+\phi+\mu_3)} + \frac{\alpha\gamma\xi\mu_2(\beta(\gamma+\mu_1)-\alpha\beta_1)}{(\gamma+\mu_1)(\alpha\xi+(\epsilon+\mu_2)(\gamma+\mu_1))^2(\beta+\phi+\mu_3)}.$$

Table List of Parameters used in Model System (1)

Parameter	**Range**	**Default Values** (day^{-1})	**Description** (day^{-1})
α	10 - 100	50	Recruitment Rate into Single Population
γ	0.015 - 0.075	0.015	Marriage Rate of Single Population
ϵ	0.01 - 0.08	0.01	Divorce Rate of Married Individuals
β	0.0.007 - 0.05	0.008	Rate at which Divorced Individuals become Single again
β_1	0.00013 - 0.0004	0.00015	Interaction Rate: Divorced + Single $\rightarrow$ Married
ξ	0.0001 - 0.08	0.04	Interaction Rate: Single + Married $\rightarrow$ Divorced
μ_1	0.05 - 0.08	0.05	Mortality Rate of Single Population
μ_2	0.01 - 0.03	0.01	Mortality Rate of Married Population
μ_3	0.006 - 0.03	0.03	Mortality Rate of Divorced Population
ϕ	0.0005 - 0.005	0.004	Additional Exit (Mental Disorder) Rate from Divorced Population

Table 1: Parameter Values (Tessema et al., 2022; Duato & J'odar, 2013; Gambrah & Adzadu, 2018; Lhous et al., 1999) used in the Population Dynamics over 2000 days

Numerical Simulation

In this section, we have tried to analyse the dynamics of the Single, Married and Divorced Population numerically over time of 2000 days. We have initially considered Single Population of 10,000 people and taken both Married and Divorced Populations as zero in a particular geographical region. At first, only Single Population is found in a specific geographical area, while the other two populations remain at zero. The Single Population has

decreased exponentially from 10,000 to below 2,000 after 2000 days (Figure 1). On the other hand, the Married Population has increased by more than 4000 people over 2000 days (Figure 2). The Divorced Population has reached about 1500 people after 2000 days (Figure 3). In Figure 4, graphical representations of Single, Married and Divorced Populations together over time of 2000 days are illustrated. Here we have observed that the crossing over of the Single and Married Population (both populations are above 4000) has been happened at about 1700 days starting from initial condition. As it is natural from our model system that the Single Population has decreased and the Married Population has increased along with time.

Now we have tried to observe the effect increasing value of the dishonest interaction rate between a Single and a Married individual (ξ) on Divorced Population over various time span. We have taken three significant values of ξ as 0.00005, 0.0001 and 0.0002 respectively. In Figure 5, graphical representations of Divorced Population over time for 300 days for increasing value of ξ are exhibited. Initially, the Divorced Population has increased significantly based on the values of ξ and then it has stabilised very slowly. It is also noticed that the Divorced Population is increasing with the increasing value of ξ. In Figure 6 and Figure 7, we have taken time span for 2000 days (5.5 years approx.) and 5000 days (13.5 years approx.). It is significantly noticed that as time has increased, the Divorced Population becomes stable in nature very sharply and quickly. In all cases, the Divorced Population is stabilising around more or less 2000 people according to the values of ξ.

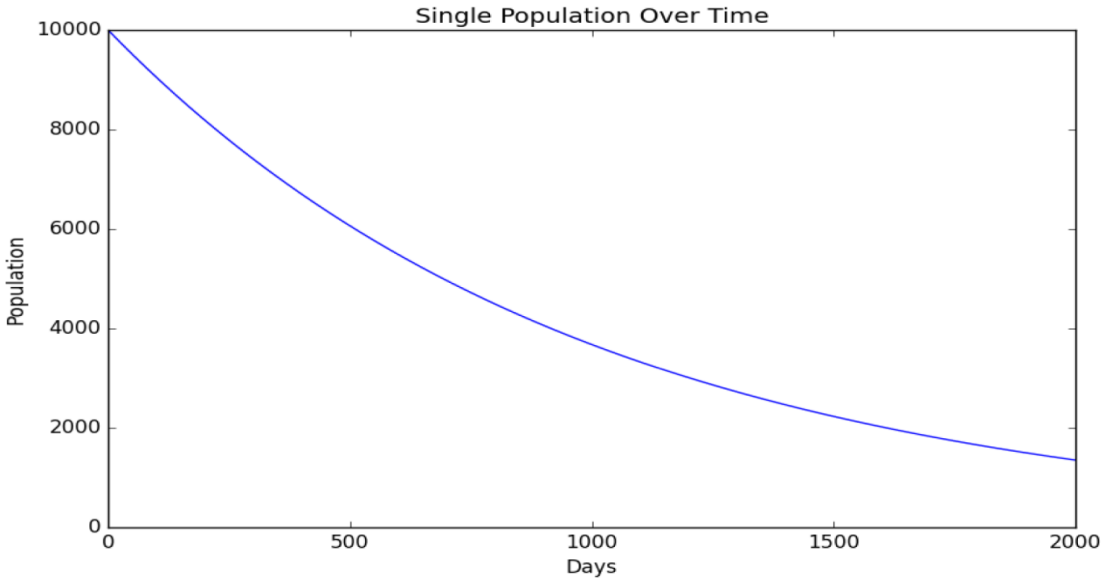

Figure 1: Graphical Representation of Single Population over time

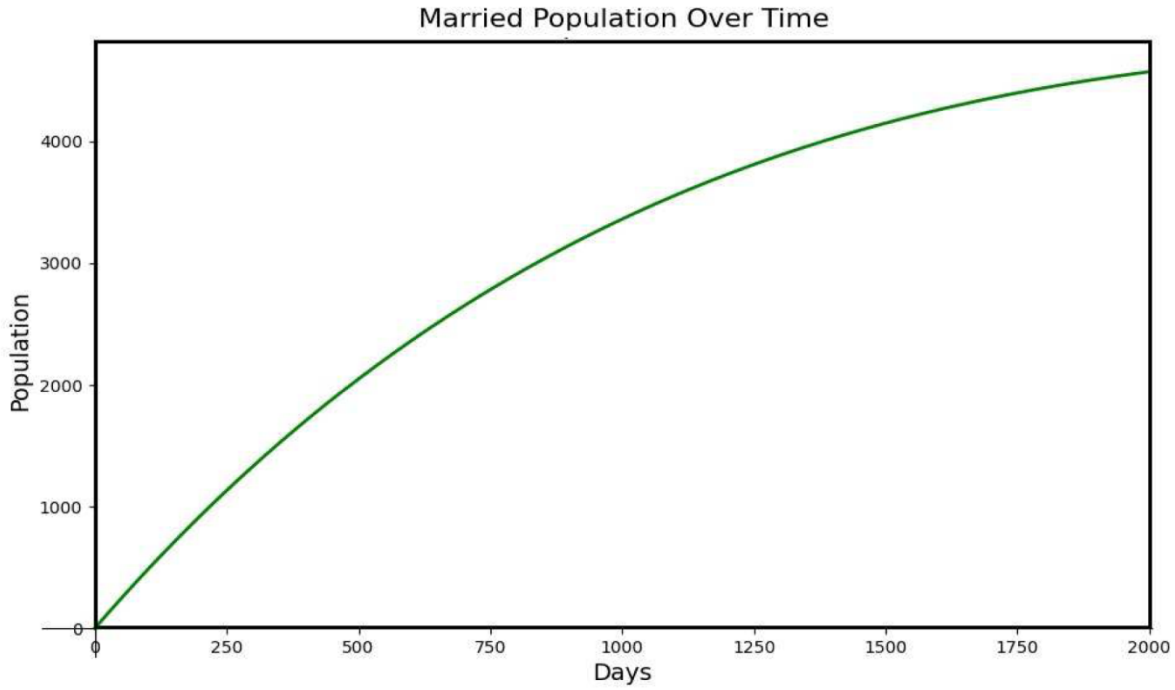

Figure 2: Graphical Representation of Married Population over time

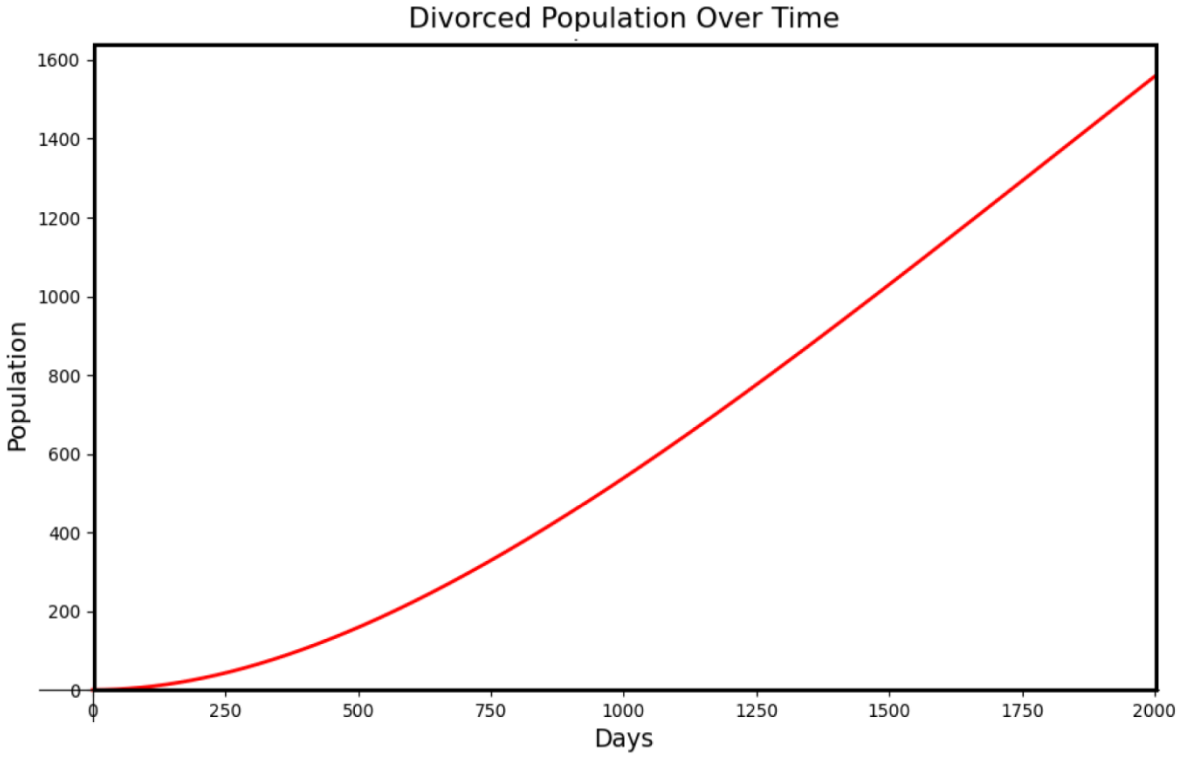

Figure 3: Graphical Representation of Divorced Population over time

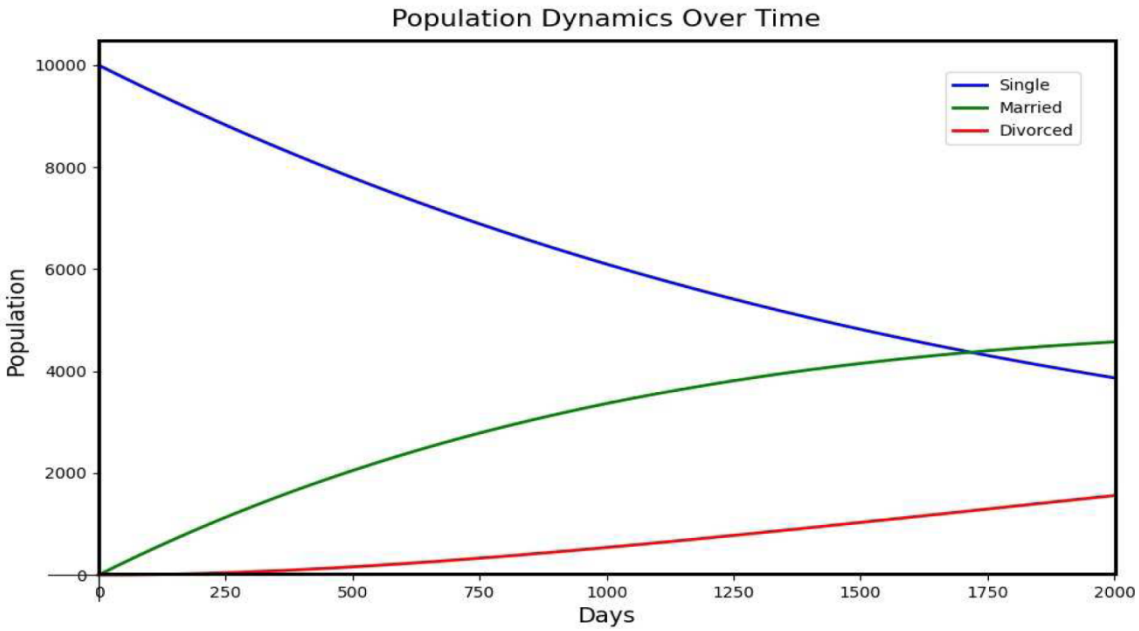

Figure 4: Graphical Representation of Single, Married and Divorced Population together over time

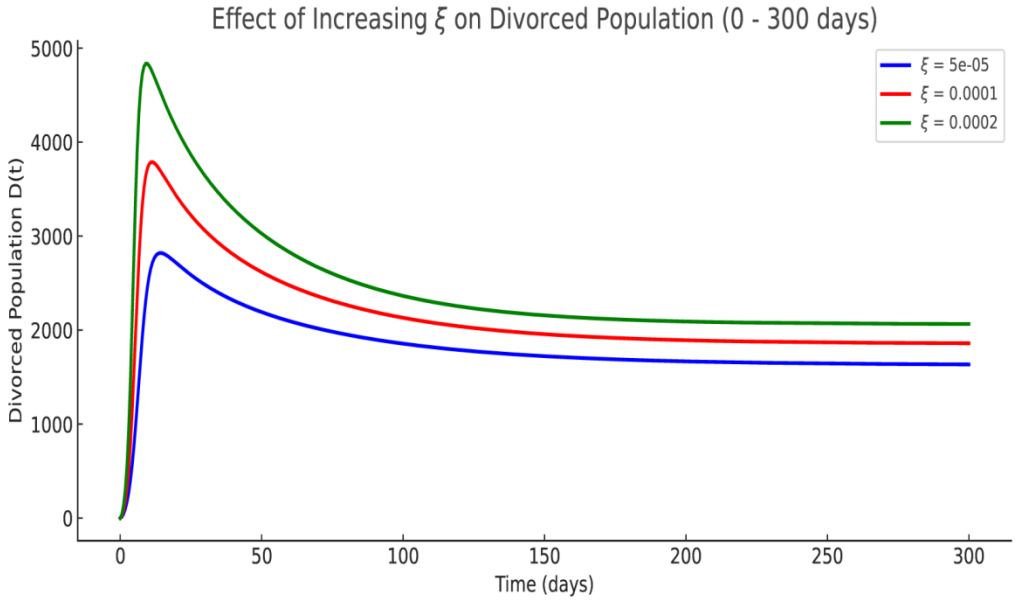

Figure 5: Graphical Representation of Divorced Population over time (300 days) for increasing value of ξ

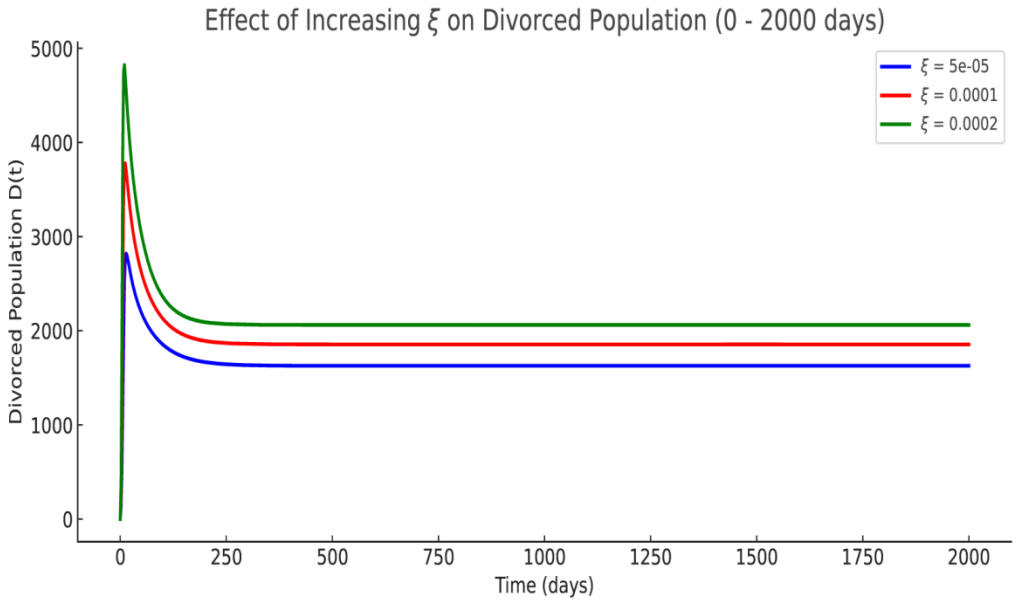

Figure 6: Graphical Representation of Divorced Population over time (2000 days) for increasing value of ξ

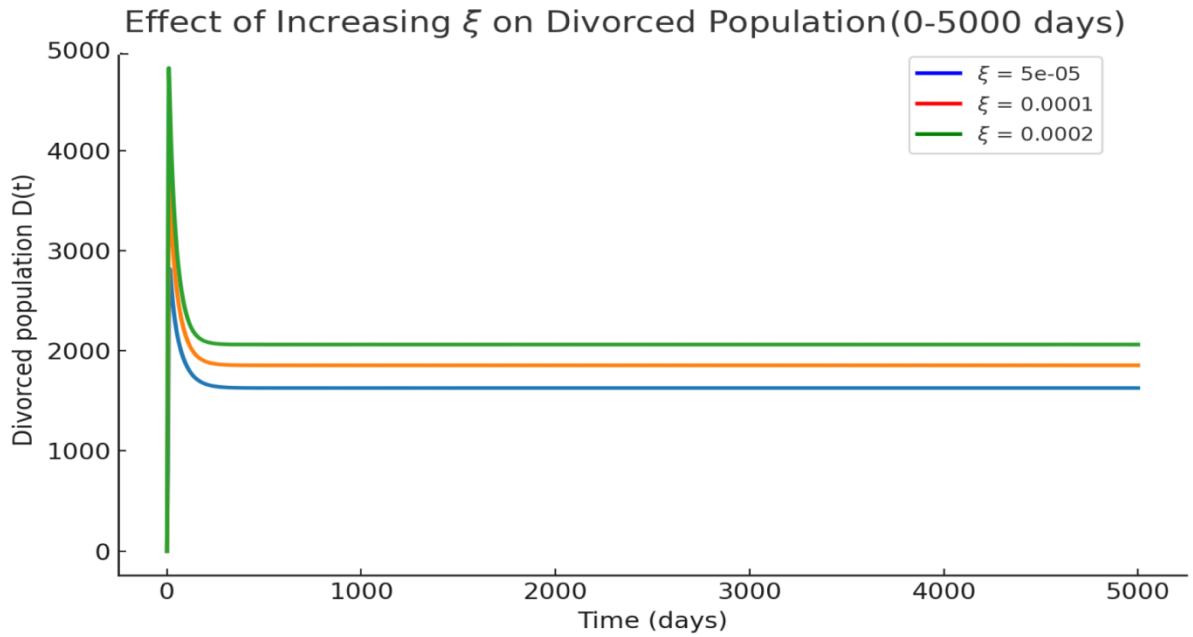

Figure 7: Graphical Representation of Divorced Population over time (5000 days) for increasing value of ξ

In Figure 8, (Case A: $R_0 > 1$, $\xi = 0.06$ $\beta = 0.04$, $\beta_1 = 0.001$) we have observed that the Divorce-free equilibrium is unstable, even with D(0) = 0 Here, it shows small inflows from Married Population, which triggers sustained growth of Divorced Population. In Figure 9, (Case B $R_0 < 1$, $\xi = 0.03$, $\beta = 0.06$, $\beta_1 = 0.001$) the Divorce-free equilibrium is stable, even with D(0) = 1500 Here Divorced Population declines to zero.

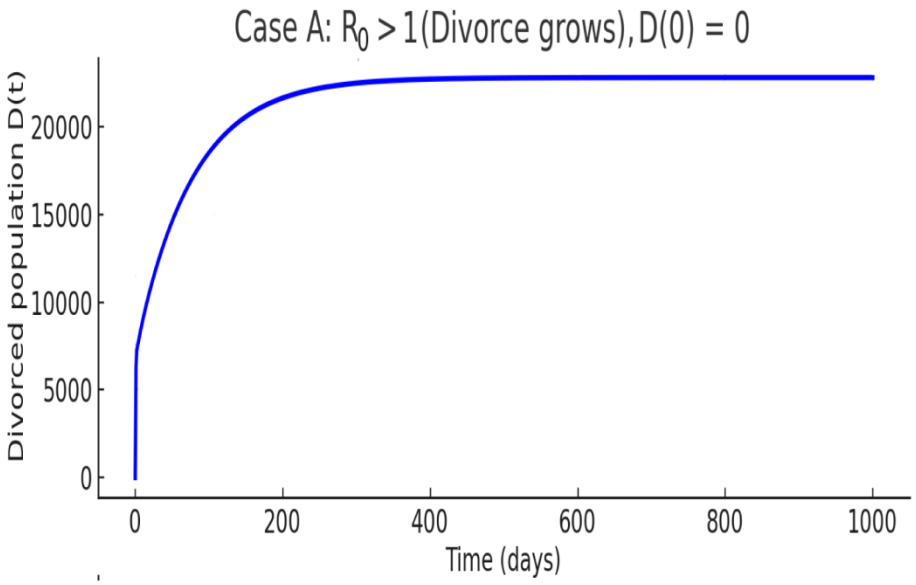

Figure 8: Graphical Representation of Divorced Population over time (1000 days) for $R_0 > 1$

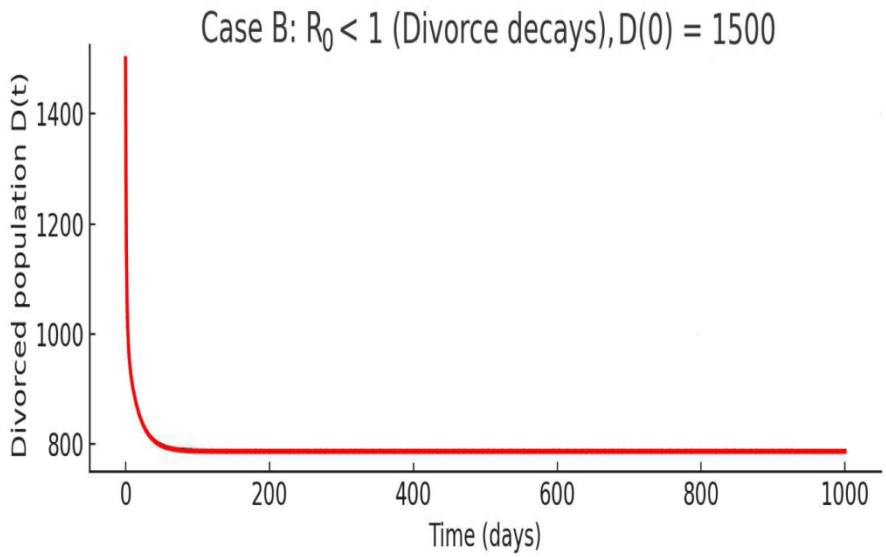

Figure 9: Graphical Representation of Divorced Population over time (1000 days) for $R_0 < 1$

In Figure 10, the Contour Plot illustrates how the Basic Reproduction Number R_0 of the Divorce Dynamical Model varies with the Contact Divorce rate ξ (vertical axis) and the rejoin rate 8 (horizontal axis). The colour shading represents R_0 values, where Yellow regions indicate $R_0 > 1$ and Purple / Blue regions indicate $R_0 < 1$ The contour line between these regions marks the threshold $R_0 = 1$ which separates two qualitatively different system behaviors. When parameters fall in the $R_0 > 1$ region (higher ξ or lower B), Divorce cases are expected to grow and persist over time, making the Divorce-free equilibrium unstable. In contrast, when parameters lie in the $R_0 < 1$ region (lower ξ or higher β), Divorce cases will decline and eventually vanish, resulting in a stable Divorce-free equilibrium. Therefore, interventions should aim to either reduce ξ (limiting conditions that trigger Divorces) or increase (facilitate reintegration) to maintain the system in the $R_0 < 1$ region.

Also, Figure 11 represents a Contour Plot of the Basic Reproduction Number R_0 for the Divorce Dynamical Model as a function of the rejoin rate β (horizontal axis) and the Contact Divorce rate ξ (vertical axis). The colored regions show different Ro values, when White dashed line is marking the critical threshold $R_0 = 1$ Points above and to the left of this line correspond to $R_0 > 1$ where Divorce cases are expected to grow and persist, making the Divorce-free equilibrium unstable. Points below and to the right of the line correspond to $R_0 < 1$ where Divorce cases will decline over time, leading to a stable Divorce-free equilibrium. Two specific parameter combinations are highlighted: ($\beta = 0.04$ $\xi = 0.08$) in Red, where $R_0 = 2$ indicates sustained growth of Divorces and ($\beta = 0.06$; $\xi = 0.03$) in Green, where $R_0 = 0.5$ indicates that Divorces will eventually vanish. This plot clearly demonstrates how adjusting β and ξ can move the system between persistence and elimination of Divorces from a particular geographical area.

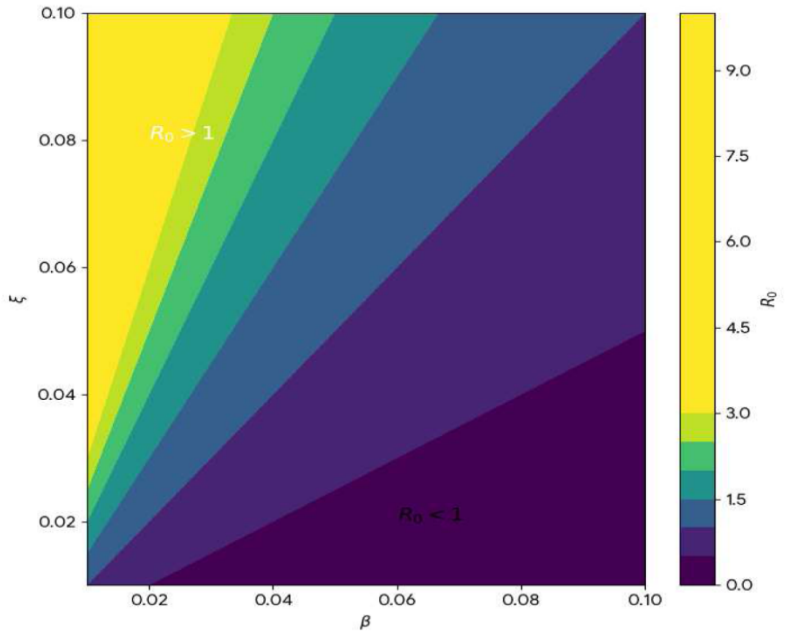

Figure 10: Contour Plot of System Efficiency R_0 as a function of ξ and β

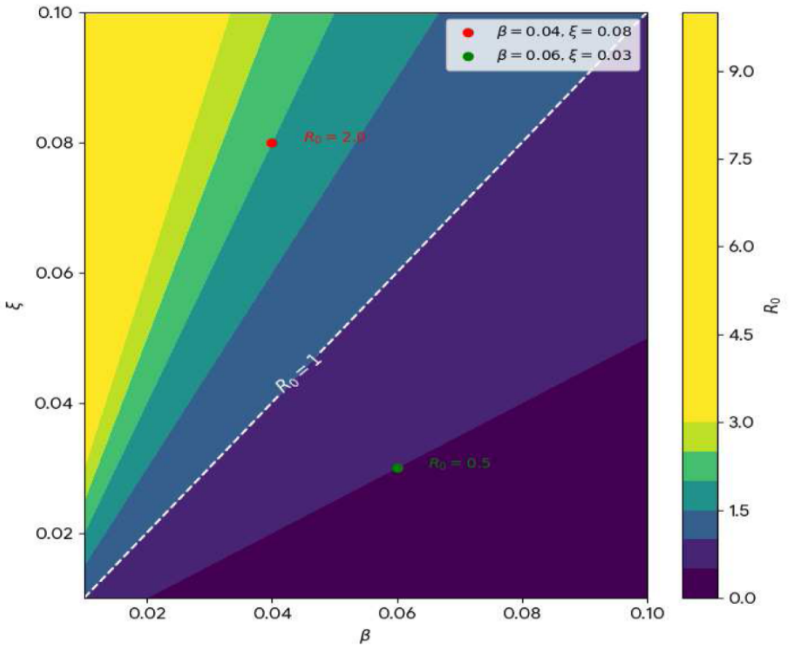

Figure 11: Contour Plot of System Efficiency R_0 as a function of ξ and β, which represents threshold condition for R_0

Conclusion

We are trying to illustrate numerically the dynamical behaviour of Single, Married and Divorced Populations over time span of 2000 days (5.5 years approx.). It is well observed from the Figure (1 and 2) that Single Population decreases as well as Married Population increases gradually as continuous Marriage has been taken place in our society. On the other hand, Divorced Population also increases starting from zero level as Divorce also occurs in our surroundings (Figure 3). A crossing over has been occurred between Single and Married Populations (Figure 1) after 1700 days (1.5 years approx.).

We have successfully established the noteworthy effect of the dishonest connection rate be-tween a Single and a Married individual on Divorced Population over various time period. The more improper connections there are, the more the rate of Divorce will increase at the beginning of a conjugal life. This unholy communication between the Single and Married couple causes more damage in the first 1 to 2 years of Marriage. As many days will pass, the detrimental impact of this dishonest contact will continue to diminish. We have noticed that the flow has been decreased significantly after 5.5 years and 13.5 years. Higher dishonest contact or lower re-join rate of the Divorced people into the Single compartment, Divorce cases are projected to raise and persevere over time, which declares the unstable Divorce-free system. On the other hand, Divorce cases will be weakening and eventually tends to disappear, resulting in a stable Divorce-free system for lower dishonest contact or higher re-join rate of the Divorced people into the Single compartment. Therefore, our focus should aim to either reduce dishonest communication that triggers Divorces or enhance reintegration of Divorced people to maintain the vigorous conjugal system. Single people are like a pile of gunpowder. A small spark can cause a huge explosion. In most cases, they end up destroying themselves and sending a family to the brink of ruin. We can conclude that a Married couple will require at least 1 to 2 years to stay away from the influence of Single people with dishonest mind. Although it is very difficult to recognize an evil person, yet we must try to spot them in order to maintain an energetic and dynamic marital relationship.

References:

Duato, R., J´odar, L. (2013). Mathematical modeling of the spread of divorce in Spain. Mathematical and Computer Modelling, 57(7 – 8), 1732–1737.

Gambrah, P. P., Adzadu, Y. (2018). Mathematical model of divorce epidemic in Ghana. International Journal of Statistics and Applied Mathematics, 2018(3(2)), 395–401.

Gweryina, R. I., Kaduna, F. S., Kura, M. Y. (2021). Qualitative analysis of a mathematical model of divorce epidemic with anti-divorce therapy. Engineering and Applied Science Letters, 4(2), 1–11.

Hugo, A., Lusekelo, E. M. (2021). Mathematical control of divorce stress among marriages. International Journal of Statistics and Applied Mathematics, 6(5), 126–136.

Karaagac, B., Owolabi, K. M. (2023). A numerical investigation of marriage divorce model: Fractal fractional perspective. Scientific African, 21.

Kleinsorge, C., Covitz, L. M. (2012). Impact of divorce on children: developmental considerations. Pediatrics in Review, 33, 147–155.

Lhous, M., Rachik, M., Laarabi, H., Abdelhak, A. Discrete mathematical modeling and optimal control of the marital status: the monogamous marriage case, Advances in Difference Equations. (1999).

Mekonnen, Y., Kassa, K., Ayalew, M. (2019). Prevalence, causes and consequences of divorce in Bahir Dar city, Ethiopia. African Journal of Social Work, 9(1).

Mikolai, J., Kulu, H. (2018). Short and long-term effects of divorce and separation on housing tenure in England and Wales. Population Studies, 72, 17–39.

Moon, M. (2011). The Effects of Divorce on Children: Married and Divorced Parents' Perspectives. Journal of Divorce and Remarriage, 52(5), 344–349.

Murray, J. D. Mathematical Biology: I, An Introduction. (2007). Springer.

Padder, A., Qureshi, S., Soomro, A., Shaikh, F., Hincal, E., Chang, C. W. (2025). Evaluating divorce dynamics through ODE modeling and statistical hypothesis testing. Discover Applied Sciences, 7(592).

Shah, J., Khan, U., Alqarni, M. M., Shah, K., Mahmoud, E. E. (2025). Dynamics of Divorce Epidemic: A Nonlinear Mathematical Model with Stochastic Influences and Behavioral Factors. Mathematical Methods in the Applied Sciences, 48(13), 12979–12988.

Tessema, H., Haruna, I., Osman, S., Kassa, E. (2022). A Mathematical Model Analysis of Marriage Divorce. Communications in Mathematical Biology and Neuroscience, 2022(15).

Tessema, H., Mengistu, Y., Kassa, E. (2022). Analysis of the Mitigation Strategies for Marriage Divorce: from Mathematical Modeling Perspective. Journal of Applied Mathematics and Informatics, 40(5 - 6), 857-871.